14.3: Using Trigonometric Identities

Reciprocal Identities:

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sin\theta = \frac{1}{\csc\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \cos\theta = \frac{1}{\sec\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

Tangent and Cotangent Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

**We can rearrange these if needed

$$\sin^2 + \cos^2 = 1$$

$-\sin^2$ $-\cos^2$ $-\cos^2$ $-\sin^2$

* $\sin^2 = 1 - \cos^2$

* $\cos^2 = 1 - \sin^2$

$$\tan^2 + 1 = \sec^2$$

$$-\tan^2 \quad -1 \qquad\qquad -1 \quad -\tan^2$$

$$*\ \tan^2 = \sec^2 - 1$$

$$*\ \sec^2 - \tan^2 = 1$$

Example: Simplify the left side of the equation to match the right side of the equation.

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

a.) $\cos\theta \tan\theta = \sin\theta$

$$\frac{\cancel{\cos\theta}}{1}\ \frac{\sin\theta}{\cancel{\cos\theta}}$$

$\sin\theta$

b.) $(\sec x - 1)(\sec x + 1) = \tan^2 x$

$\sec^2 x + \cancel{\sec x} - \cancel{\sec x} - 1$

$\sec^2 x - 1$

$* \quad \tan^2 x + 1 = \sec^2 x$

$\quad\quad\quad\quad\;\; -1 \quad\quad -1$

$\tan^2 x = \underline{\sec^2 x - 1}$

$\Rightarrow \tan^2 x$

$$\frac{1}{\sin^2 A} - \frac{1}{\tan^2 A} = 1$$

* $\tan^2 = \frac{\sin^2}{\cos^2}$

$$\frac{1}{\sin^2 A} - \frac{1}{\frac{\sin^2 A}{\cos^2 A}}$$

$\frac{1}{1} \cdot \frac{\cos^2}{\sin^2}$

$$\frac{1}{\sin^2 A} - \frac{\cos^2 A}{\sin^2 A}$$

* $\sin^2 A + \cos^2 A = 1$

$-\cos^2 \quad -\cos^2$

$\sin^2 A = 1 - \cos^2 A$

$$\frac{1 - \cos^2 A}{\sin^2 A}$$

$$\frac{\sin^2 A}{\sin^2 A}$$

$$= 1$$

$* \quad \sec x = \frac{1}{\cos x}$

d.) $(1 - \cos x)(1 + \sec x)(\cos x) = \sin^2 x$

$(1 - \cos x)\left(1 + \frac{1}{\cos x}\right)(\cos x)$

$\frac{\cos}{\cos}$

$(1 - \cos x)(\cos x + 1)$

$\cancel{\cos x} + 1 - \cos^2 x - \cancel{\cos x}$

$1 - \cos^2 x$

$$\boxed{\begin{array}{l} * \ \sin^2 x + \cos^2 x = 1 \\ \quad\quad\ \ -\cos^2 \quad\quad\ \ -\cos^2 \\ \sin^2 x = 1 - \cos^2 x \end{array}}$$

$= \sin^2 x$

e.) $$\frac{\tan^2\theta}{\sec\theta+1}+1 = \sec\theta$$

$* \tan^2 = \sec^2 - 1$

$$\frac{\sec^2\theta - 1}{\sec\theta + 1} + 1$$

$x^2 - 1$

$(x+1)(x-1)$

$$\frac{\cancel{(\sec\theta+1)}(\sec\theta-1)}{\cancel{\sec\theta+1}} + 1$$

$\sec\theta - 1 + 1$

$= \sec\theta$