

## 14.3: Using Trigonometric Identities

Reciprocal Identities:

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sin \theta = \frac{1}{\csc \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{1}{\cot \theta}$$

## Tangent and Cotangent Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

## Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

\*\*We can rearrange these if needed

$$\sin^2 + \cos^2 = 1$$

*(Note: In the original image, there are handwritten annotations below the terms: a green '-sin^2' under the first sin^2, a blue '-cos^2' under the second cos^2, and a blue '-cos^2' and a green '-sin^2' under the '1'.)*

$$* \sin^2 = 1 - \cos^2$$

$$* \cos^2 = 1 - \sin^2$$

$$\tan^2 + 1 = \sec^2$$

$-\tan^2$        $-1$        $-\tan^2$

$$*\tan^2 = \sec^2 - 1$$

$$*\sec^2 - \tan^2 = 1$$

Example: Simplify the left side of the equation to match the right side of the equation.

a.)  $\cos \theta \tan \theta = \sin \theta$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\frac{\cancel{\cos \theta}}{1} \cdot \frac{\sin \theta}{\cancel{\cos \theta}}$$

$$\sin \theta$$

$$b.) (\sec x - 1)(\sec x + 1) = \tan^2 x$$

$$\sec^2 x + \cancel{\sec x} - \cancel{\sec x} - 1$$

$$\sec^2 x - 1$$

$$* \tan^2 x + 1 = \sec^2 x$$

$$\tan^2 x = \sec^2 x - 1$$

$$\Rightarrow \tan^2 x$$

$$\frac{1}{\sin^2 A} - \frac{1}{\tan^2 A} = 1$$

\*  $\tan^2 = \frac{\sin^2}{\cos^2}$

$$\frac{1}{\sin^2 A} - \frac{\frac{1}{\sin^2 A}}{\cos^2 A} \rightarrow \frac{1}{1} \cdot \frac{\cos^2}{\sin^2}$$

$$\frac{1}{\sin^2 A} - \frac{\cos^2 A}{\sin^2 A}$$

\*  $\sin^2 A + \cos^2 A = 1$   
 $-\cos^2 A \quad -\cos^2 A$

$$\frac{1 - \cos^2 A}{\sin^2 A}$$

$\sin^2 A = 1 - \cos^2 A$

$$\frac{\sin^2 A}{\sin^2 A} = 1$$



\*  $\sec x = \frac{1}{\cos x}$

d.)  $(1 - \cos x)(1 + \sec x)(\cos x) = \sin^2 x$

$$(1 - \cos x) \left( 1 + \frac{1}{\cos x} \right) (\cos x)$$

$$(1 - \cos x)(\cos x + 1)$$

$$\cancel{\cos x} + 1 - \cos^2 x - \cancel{\cos x}$$

$$\rightarrow 1 - \cos^2 x$$

$$* \sin^2 + \cos^2 = 1$$

$$\sin^2 = 1 - \cos^2$$

$$= \sin^2 x$$

e.)  $\frac{\tan^2 \theta}{\sec \theta + 1} + 1 = \sec \theta$  \*  $\tan^2 = \sec^2 - 1$

$$\frac{\sec^2 \theta - 1}{\sec \theta + 1} + 1$$

$x^2 - 1 = (x+1)(x-1)$

$$\frac{(\cancel{\sec \theta + 1})(\sec \theta - 1)}{\cancel{\sec \theta + 1}}$$

$$\sec \theta - 1 + 1$$

$$= \sec \theta$$