7.4: Evaluate Logarithms and Graph their Functions

*We know that: $2^2 = 4$

$2^3 = 8$

However what value of x does: $2^x = 6$

-will it be a whole number?

We can use a logarithm to change this to a form easy to calculate

Definition of Logarithm with Base b

$$\log_b y = x \longrightarrow b^x = y$$

Example 1: Rewrite in exponential form

a. $\log_3 81 = 4$ $3^4 = 81$

b. $\log_7 7 = 1$ $7^1 = 7$

c. $\log_{14} 1 = 0$ $14^0 = 1$

d. $\log_{1/2} 32 = -5$

$$\frac{1}{2}^{-5} = 32$$

$$\frac{2}{1}^{5} = 32$$

$$\frac{32}{1}$$

Example 2: Evaluate the logrithm without a calculator

$\log_{10}$ is written simply as log

a. $\log_2 32 = x$ $\quad 2^? = 32 \quad x = 5$

b. $\log_{27} 3 = x$ $\quad 27^? = 3 \quad \sqrt[3]{27} = 3 \quad x = \frac{1}{3}$

c. $\log_3 9 = x$ $\quad 3^x = 9 \quad x = 2$

d. $\log_2 8 = x$ $\quad 2^x = 8 \quad x = 3$

e. $\log_{16} \frac{1}{4} = x$

$16^x = \frac{1}{4}$

$16^{-\frac{1}{2}} = \frac{1}{4}$ $\quad \boxed{x = -\frac{1}{2}}$

$\frac{1}{16^{1/2}} = \frac{1}{4}$

***Important Info:

What does a negative exponent do?

$$2^{-1} = \frac{1}{2^1} = \frac{1}{2}$$

$$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$$

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

What does a fractional exponent do?

$$4^{1/2} = \sqrt[2]{4}^{1} = 2$$

$$8^{1/3} = \sqrt[3]{8} = 2$$

Natural Logs

$\log_e x \longrightarrow \ln x$

LN x

-We don't write it as $\log_e$ we write it as ln

- ln x and e^x are inverses of each other

Example 3: Simplify

a. $8^{\log_8 x}$ $\qquad \log_8 x \rightarrow 8^x$

8ˣ and $\log_8$ are inverses of each other

What happens when you combine inverses?

for example: what is 2x times $\frac{1}{2}$

using this lets solve: $\cancel{8}^{\cancel{\log_8} x} = x$

b. $\log_7 7^{-3x} = -3x$

$2^6 = 64$

c. $\log_2 64^x$

we need to make the 64 another number with base of 2

$\cancel{\log_2 2}^{6x} = 6x$

d. $\cancel{e}^{\ln 20} = 20$

Example 4: Find the inverse

What do we have to do first to find the inverse?

↳ switch $x \to y$

a. $y = 4^x$

$x = 4^y$

$\log_4 x = \cancel{\log_4 4}^y$

$y = \log_4 x$

$4^x \to \log_4 x$

$f^{-1}(x) = \log_4 x$

b. $y = \ln(x - 5)$

$x = \cancel{\ln}(y-5)$

$e \quad e$

$e^x = y - 5$

$+5 \quad +5$

$y = e^x + 5$

c. $y = \log_6 (x + 1) - 6$

$x = \log_6(y+1) - 6$

$+6 \quad +6$

$6^{x+6} = \cancel{6^{\log_6}}(y+1)$

$6^{x+6} = y + 1$

$-1 \quad -1$

$y = 6^{x+6} - 1$

d. $y = 4\log_3(3x + 7)$

$$\frac{x}{4} = \frac{4\log_3(3y+7)}{4}$$

$$3^{\frac{x}{4}} = \cancel{3^{\log_3}}(3y+7)$$

$$3^{\frac{x}{4}} = 3y + 7$$
$$-7 \qquad -7$$

$$\frac{3^{\frac{x}{4}} - 7}{3} = \frac{3y}{3}$$

$$y = \frac{3^{x/4} - 7}{3}$$

What does the graph look like?

$y = \log x$ → Parent

there isn't a base written, what number can we put there?

$y = \log_{10} x$

what happens when we plug in numbers?

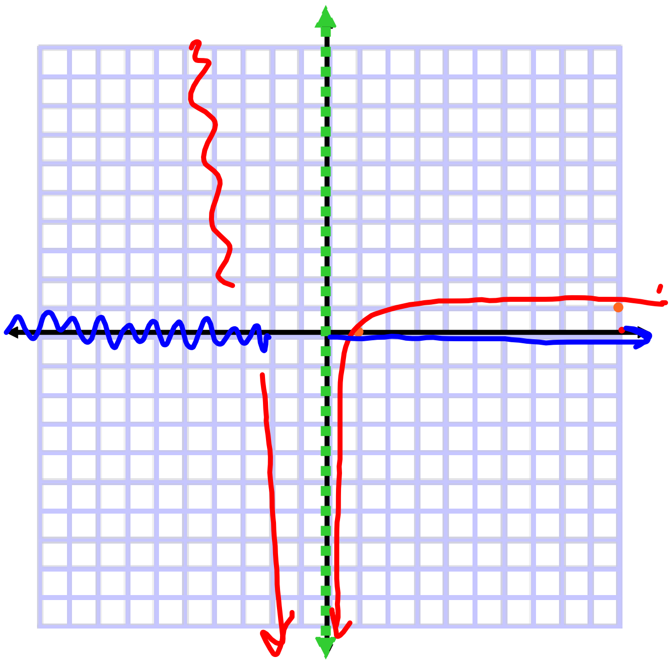
-2	error
-1	error
0	error
1	0
2	.301
⋮	
10	1

why are these whole numbers?

$\log_{10} 1 = ?$ ⟶ $10^? = 1$

$\log_{10} 10 = ?$ ⟶ $10^? = 10$

this is the pattern we will use for all graphing



(1, 0)
(10, 1)

Domain: $(0, \infty)$

Range:

$(-\infty, \infty)$

Foldable (Graphing Logarithmic Functions)

left/ right

$y = \log_b(x - h) + k$

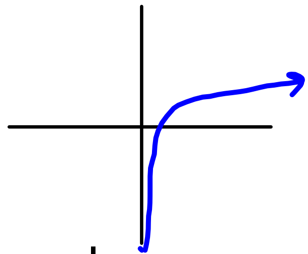
up/ down

* switch h

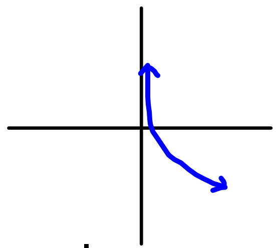
Asymptote: $x = h$

$y = \log_b x$

Points: (1, 0)
(b, 1)



$\log_b x$
$b > 1$



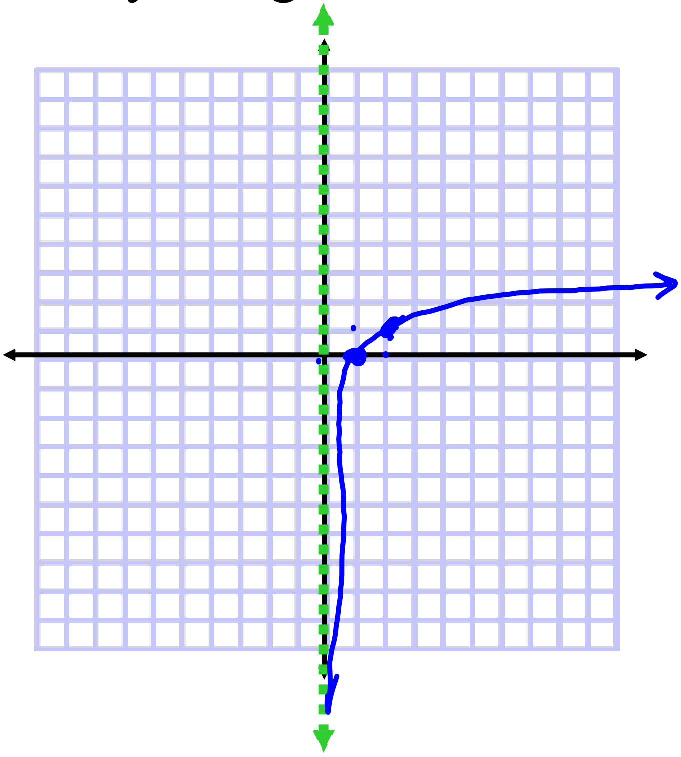
$\log_b x$
$0 < b < 1$

Domain: (h, ∞)

Range: $(-\infty, \infty)$

Example 5: Graph $\log_2 x$

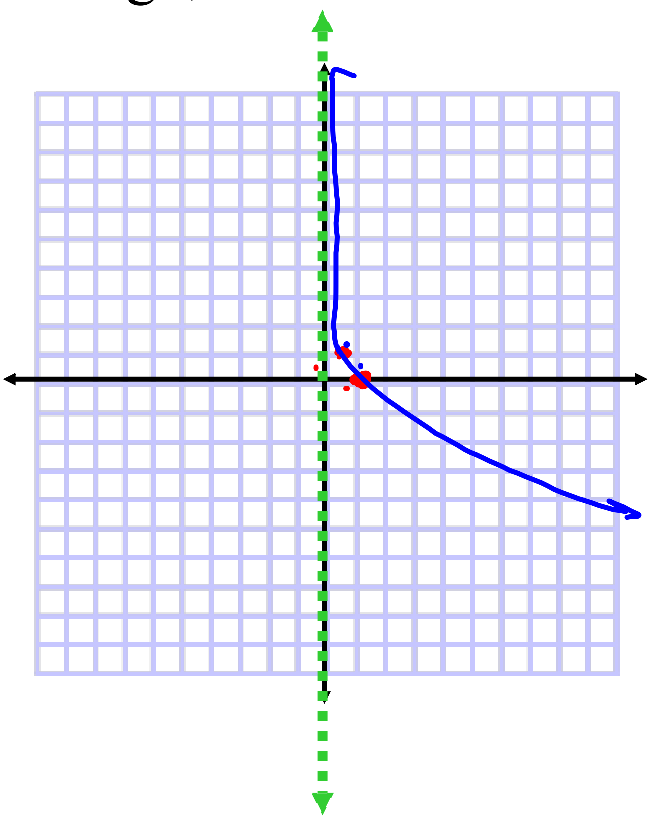
a. $y = \log_2 x$



(1, 0)
(2, 1)

Domain: $(0, \infty)$
Range: $(-\infty, \infty)$
Asymptote: $x = 0$
Shift: None

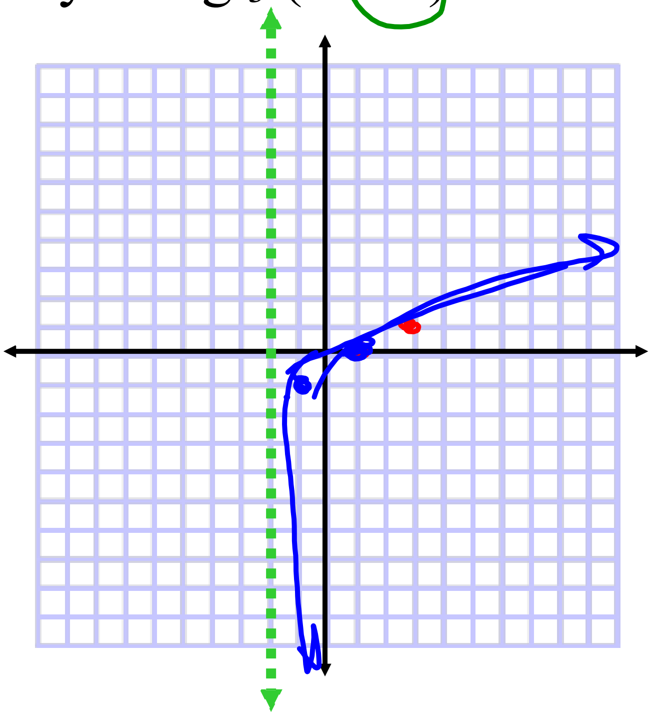
b. $y = \log_{1/2} x$



(1, 0)
($^1/_2$, 1)

Domain: $(0, \infty)$
Range: $(-\infty, \infty)$
Asymptote: $x = 0$
Shift: None

c. $y = \log_3 (x + 2) - 1$



(1, 0)
(3, 1)

Domain: $(-2, \infty)$
Range: $(-\infty, \infty)$
Asymptote: $x = -2$
Shift:

left 2
Down 1