3.) $y = \log_4(4x^5)$

$4^x = \cancel{\log_4}(4y^5)$

$\frac{4^x}{4} = \frac{4y^5}{4}$

$\sqrt[5]{\frac{4^x}{4}} = \sqrt[5]{y^5}$

$y = \sqrt[5]{\frac{4^x}{4}}$

5.) $x = \log_{\frac{1}{3}}(y-2) - 7$

$\quad +7 \qquad\qquad\qquad\qquad +7$

$\frac{1}{3}^{x+7} = \frac{1}{3}^{\log_{\frac{1}{3}}(y-2)}$

$\frac{1}{3}^{(x+7)} = y - 2$

$\quad +2 \qquad\quad +2$

$y = \frac{1}{3}^{(x+7)} + 2$

10.) $y = \log_{10}(x \overset{+1}{-1}) + 3$

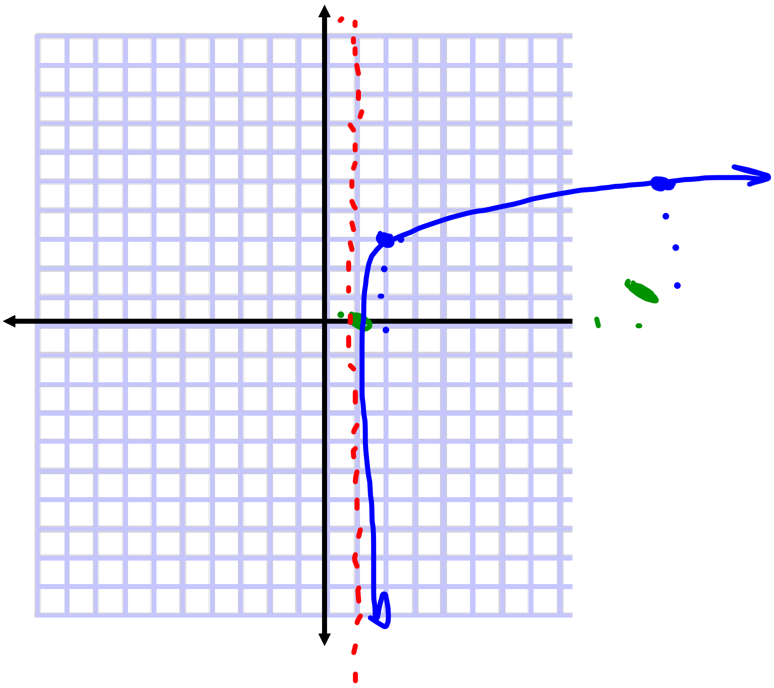
$(1, 0)$

$(10, 1)$

A: $x = 1$

Shift: Right 1

Up 3



D: $(1, \infty)$

R: $(-\infty, \infty)$

16) The function s = 0.159 + 0.118(log d) gives the slope *s* of a beach in terms of the average diame (in millimeters) of sand particles on the beach. Find the iverse of this function. Then use the inv to estimate the average diameter of the sand particles on a beach with a slope of 0.2 mm.

$$s = .159 + .118 \log d$$

$$.2 = .159 + .118 \log d$$

$$\frac{0.041}{.118} = \frac{.118 \log d}{.118}$$

$$10^{.347} = 10^{\log_{10} d}$$

$$d = 10^{.347}$$

$$d = 2.23 \text{ mm}$$

7.5: Properties of Logarithms

https://www.youtube.com/watch?v=VRzH4xB0GdM

What is a log?

Logarithms find the cause for an effect, i.e the input for some output

A common "effect" is seeing something grow, like going from $100 to $150 in 5 years. How did this happen? We're not sure, but the logarithm finds a possible cause: A continuous return of ln(150/100) / 5 = 8.1% would account for that change. It might not be the actual cause (did all the growth happen in the final year?), but it's a smooth average we can compare to other changes.

Logarithms put numbers on a human-friendly scale.

Large numbers break our brains. Millions and trillions are "really big" even though a million seconds is 12 days and a trillion seconds is 30,000 years. It's the difference between an American vacation year and the entirety of human civilization.

The trick to overcoming "huge number blindness" is to write numbers in terms of "inputs" (i.e. their power base 10). This smaller scale (0 to 100) is much easier to grasp:

- power of 0 = 10^0 = 1 (single item)
- power of 1 = 10^1 = 10
- power of 3 = 10^3 = thousand
- power of 6 = 10^6 = million
- power of 9 = 10^9 = billion
- power of 12 = 10^{12} = trillion
- power of 23 = 10^{23} = number of molecules in a dozen grams of carbon
- power of 80 = 10^{80} = number of molecules in the universe

In computers, where everything is counted with bits (1 or 0), each bit has a doubling effect (not 10x). So going from 8 to 16 bits is "8 orders of magnitude" or $2^8 = 256$ times larger. (These bit sizes refers to the amount of memory available, not the processor speed). Going from 16 to 32 bits means 16 orders of magnitude, or $2^{16} \sim 65{,}536$ times larger.

Isn't "16 extra bits of memory" better than "65,536 times more memory?".

Foldable (Change of Base)

<u>Change of Base</u> $\log_c a = \dfrac{\log a}{\log c}$ $\log_e = \ln$	<u>Product Property</u> $\log_b mn = \log_b m + \log_b n$ <u>Quotient Property</u> $\log_b \frac{m}{n} = \log_b m - \log_b n$ <u>Power Property</u> $\log_b m^n = n\log_b m$

Ex) Evaluate. Round to the nearest thousandth.

A) $\log_4 5$

Change of Base

$$\log_c a = \frac{\log_b a}{\log_b c}$$

$$\frac{\log 5}{\log 4} \approx 1.161$$

B) $\log_4 21$

$$\frac{\log 21}{\log 4} = 2.196$$

C) $\ln 8$ $= 2.079$

$\log_e 8$

Ex) Expand

A) $\log_3(4x)$

$\log_3 4 + \log_3 x$

B) $\log_3 x^4$

$4 \cdot \log_3 x$

C) $\log_4\left(\frac{x}{3y}\right)$

$\log_4 x - \log_4 3 - \log_4 y$

D) $\log_2\sqrt{x^3yz^4}$

$$\log_2(x^3yz^4)^{\frac{1}{2}}$$

$$\frac{1}{2}\left(\log_2(x^3yz^4)\right)$$

$$\frac{1}{2}\left(\log_2 x^3 + \log_2 y + \log_2 z^4\right)$$

$$\frac{1}{2}\left(3\log_2 x + \log_2 y + 4\log_2 z\right)$$

Ex) Condense

A) $5\log_4 a + 12\log_4 b + 3\log_4 c$

$\log_4 a^5 + \log_4 b^{12} + \log_4 c^3$

$\log_4 (a^5 b^{12} c^3)$

B) $2\log_3 x - 3\log_3 11 - 6\log_3 u$

$\log_3 x^2 - \log_3 11^3 - \log_3 u^6$

$\log_3 \dfrac{x^2}{11^3 u^6}$

Application:

For a sound with intensity I (in watts per square meter), the loudness $L(I)$ of the sound (in decibels) is given by the function:

$$L(I) = 10 \log \frac{I}{I_0}$$

where I_o is the intensity of a barely audible sound (about 10^{-12} watts per square meter). Find the decibel level of sound made at a rock concert that has a sound intensity of 10^{-1} W/m^2.

$I_0 = 10^{-12}$

$= 10 \log \frac{10^{-1}}{10^{-12}}$

10^{-1+12}

$10 \cancel{\log 10^{11}}$

$10 \cdot 11$

110 Decibels

HW: 7.5 Worksheet