

## 8.3: Graph General Rational Functions

Example:  $y = \frac{-x^2 - 3x}{x^2 + x - 2} = \frac{-x(x+3)}{(x+2)(x-1)}$

Step 1:

Simplify numerator and denominator then set denominator equal to zero & solve to find the Vertical Asymptote. (VA)

$$(x+2)(x-1) = 0 \quad \underline{x = -2, 1}$$

Step 2:

Find the Horizontal Asymptote: (HA)

A) if  $D > N$ :

HA:  $y = 0$

$D =$  degree of the denominator

B) if  $D = N$ :

HA:  $\frac{\text{Leading coeff } N}{\text{Leading coeff } D} = \frac{-1}{1}$

$N =$  degree of the numerator

C) if  $D < N$

HA: NONE

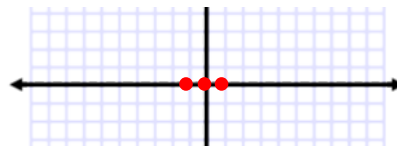
$D = 2$   $N = 2$   
So;  $D = N$   
 $y = -1$

Step 3:

Find points.

A) If no VA:

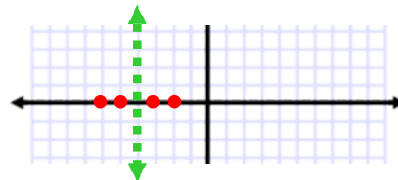
-plug in -1, 0, 1



B) If one VA:

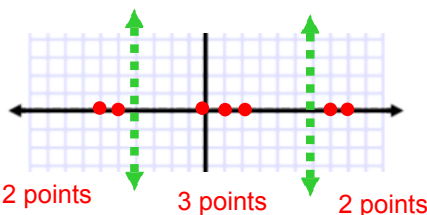
-find 2 pts. on each side of the VA

4 pts total



C) If 2 VA's:

- need 7 points



Ex) Graph

A)  $y = \frac{6}{x^2 + 1}$

VA: None

HA:  $y = 0$

Domain:  $(-\infty, \infty)$

Step 1:

$$x^2 + 1 = 0$$

$$\sqrt{x^2} = \sqrt{-1}$$

$$x = \pm i$$

Step 2:

$$D = 2$$

$$N = 0$$

$$D > N$$

Step 3:

1	3
0	6
1	3

$$\frac{6}{x^2 + 1}$$

$$\frac{6}{1+1}$$

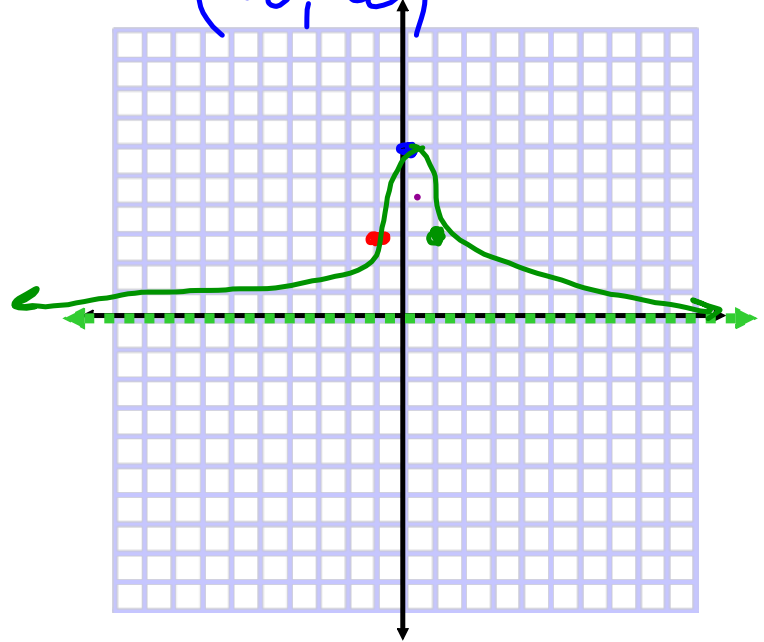
$$\frac{6}{0^2 + 1}$$

$$= \frac{6}{1}$$

$$\frac{6}{x^2 + 1}$$

$$\frac{6}{1+1}$$

$$= \frac{6}{2}$$



Range:  $(0, 6]$

$$B) y = \frac{x^2 + 3x - 4}{x - 2}$$

$$y = \frac{(x+4)(x-1)}{x-2}$$

$$VA: x = 2$$

HA: NONE

Domain:

Step 1:

$$x - 2 = 0$$

$$x = 2$$

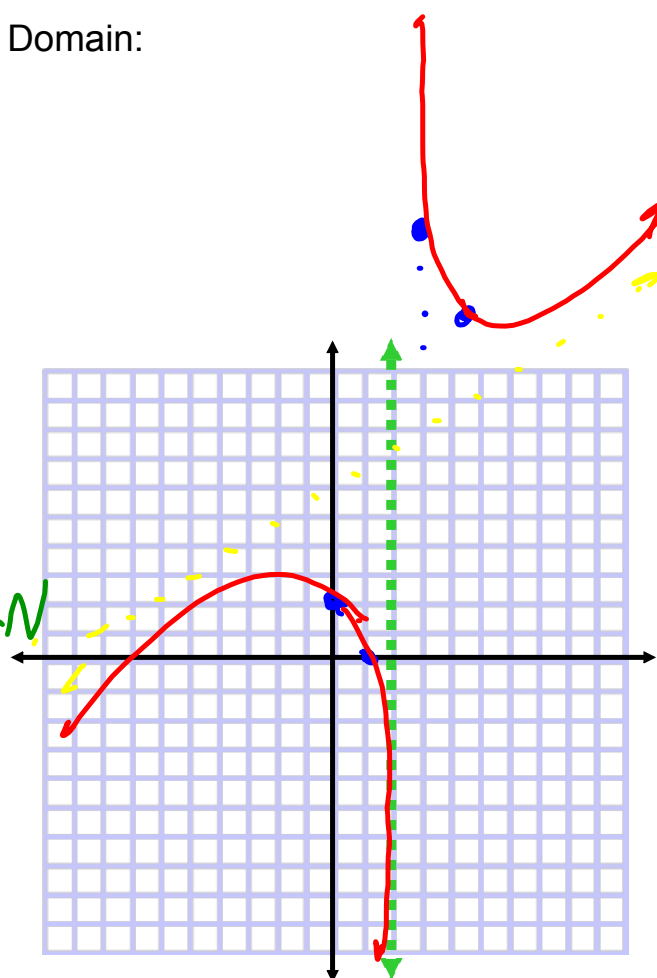
Step 2:

$$D = 1 \quad \text{so; } D < N$$

$$N = 2$$

Step 3:

$$\begin{array}{r|l} 0 & 2 \\ 1 & 0 \\ 3 & 14 \\ 4 & 12 \end{array}$$



$$C) y = \frac{-x^2}{x^2 - 4}$$

$$y = \frac{-x^2}{(x+2)(x-2)}$$

$$VA: x = 2 \text{ \& } x = -2$$

$$HA: y = -1$$

Domain:

Step 1:

$$x+2=0$$

$$x-2=0$$

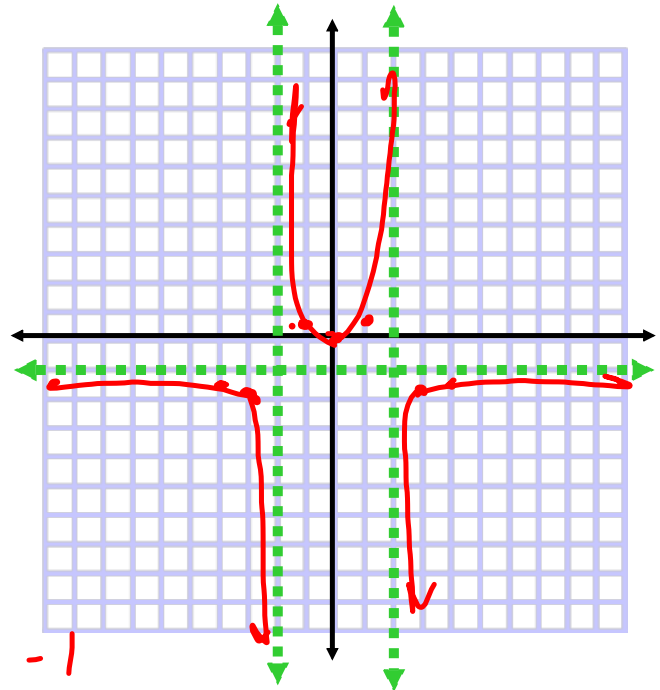
Step 2:

$$D=2$$

$$N=2$$

$$S_0; D=N$$

$$\frac{\text{leading coef } N}{\text{leading coef } D} \Rightarrow \frac{-1}{1} = -1$$



Step 3:

-4	-1.33
-3	-1.8
-1	.33
0	0
1	.33
3	-1.8
4	-1.33

D)

Example:  $y = \frac{-x^2 - 3x}{x^2 + x - 2}$

\*From beginning

VA:  $x = -2, x = 1$

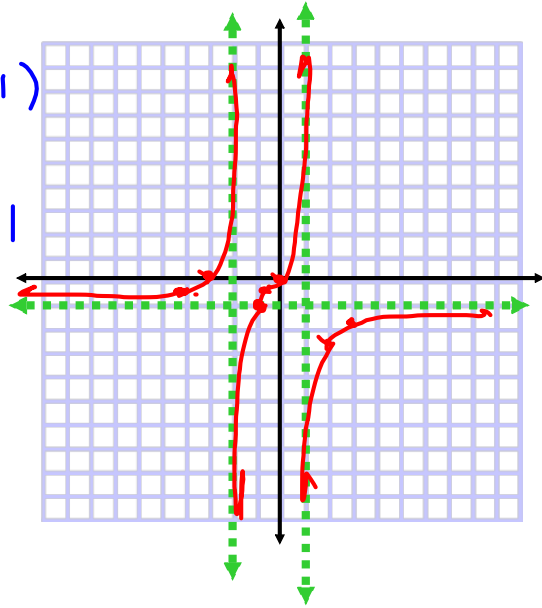
HA:  $y = -1$

D:

$$\frac{-x(x+3)}{(x+2)(x-1)}$$

Step 1:

$$\begin{aligned} x+2 &= 0 & x &= -2, 1 \\ x-1 &= 0 & & \end{aligned}$$



Step 2:

$$\begin{aligned} D &= 2 & y &= -1 \\ N &= 2 & & \\ \text{So: } D &= N & & \end{aligned}$$

Step 3:

4	-0.4
3	0
1	-1
-1/2	-1.5
3/2	0
3	-2.5
	-1.8

$$\frac{-(-\frac{1}{2})(-\frac{1}{2}+3)}{(-\frac{1}{2}+2)(-\frac{1}{2}-1)}$$

$$\frac{\frac{1}{2}(\frac{5}{2})}{(\frac{3}{2})(-\frac{3}{2})}$$

$$\frac{\frac{5}{4}}{-\frac{9}{4}}$$

$$\frac{5}{4} \cdot -\frac{4}{9} = -\frac{5}{9}$$

$$E) y = \frac{-x + 2}{x^2 - 6x + 8}$$

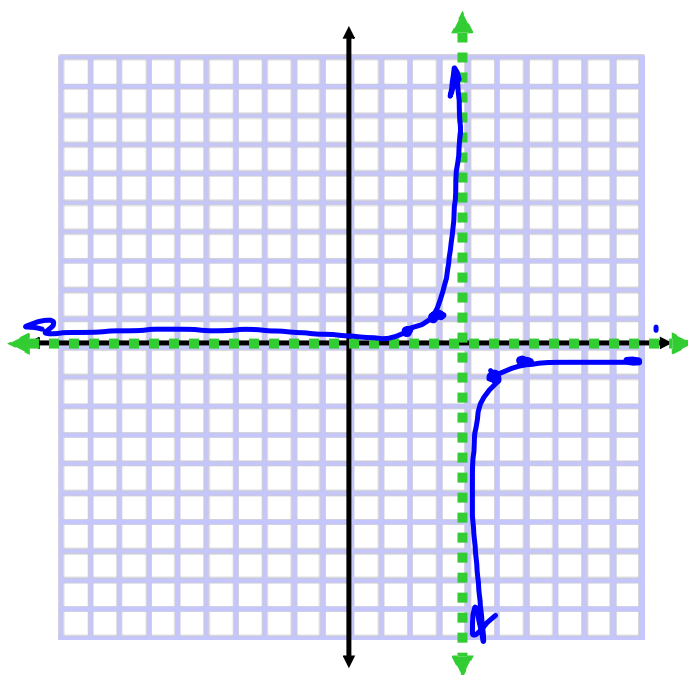
$$\frac{-\cancel{(x-2)}}{(x-4)\cancel{(x-2)}}$$

$$y = \frac{-1}{x-4}$$

$$VA: x = 4$$

$$HA: y = 0$$

$$D: x \neq 4$$



Step 1:

$$x - 4 = 0$$

$$x = 4$$

Step 2:

$$D = 1$$

$$N = 0 \quad D > N$$

Step 3:

2	1/2
3	1
5	-1
6	-1/2